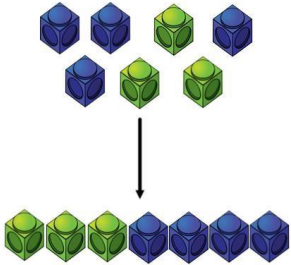
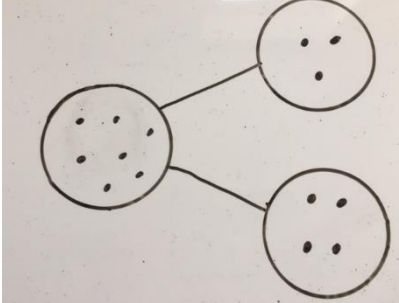
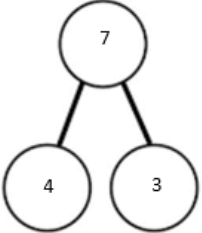
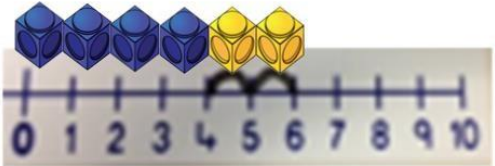
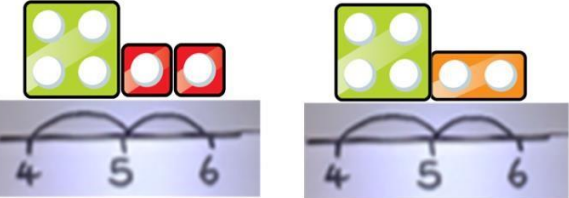
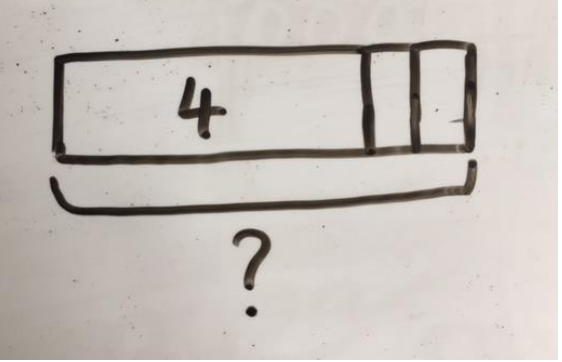

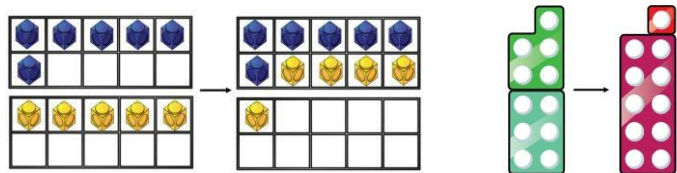


Calculation policy: Addition

Key language: 'sum, total, parts, whole, plus, add, altogether, more, is equal to' 'is the same as',
addend, aggregation, augmentation, commutative, complement, partitioning

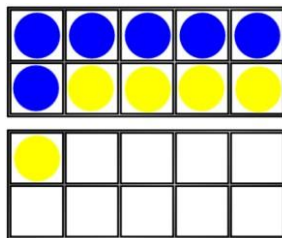
Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 
<p>Counting on using number lines using cubes or Numicon.</p>  	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.



$$6 + 5$$

Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

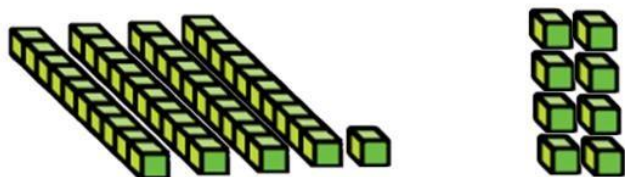
$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

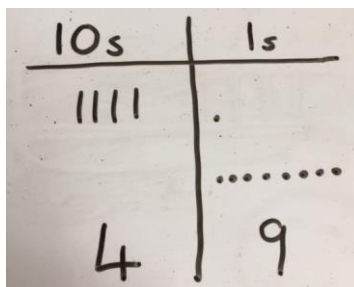
$$6 + 5 = \square + 4$$

TO + O using base 10. Continue to develop understanding of partitioning and place value.

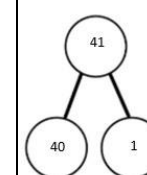
$$41 + 8$$



Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



$$41 + 8$$



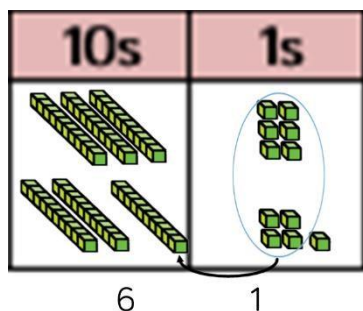
$$1 + 8 = 9$$

$$40 + 9 = 49$$

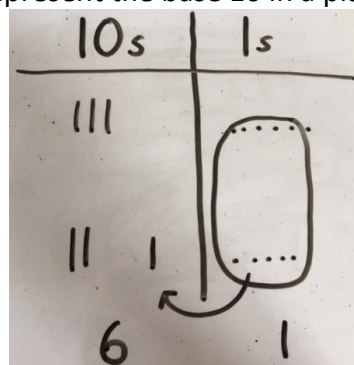
	4	1
+		8
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value.

$$36 + 25$$



Children to represent the base 10 in a place value chart.



$$43 + 24 = 67$$

$$40 + 20 = 60$$

$$3 + 4 = 7$$

$$\underline{67}$$

Or look for ways to make 10.

$$36 + 25 =$$

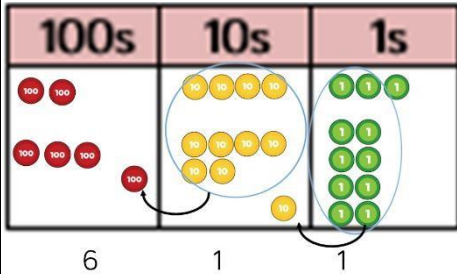
1 5

Formal method:

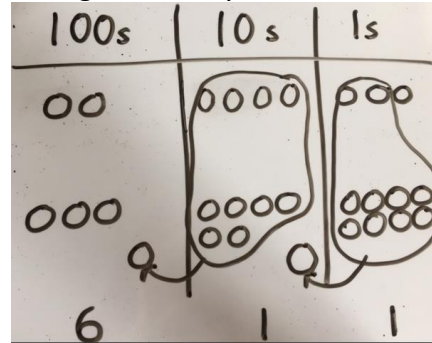
$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ 1 \end{array}$$

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



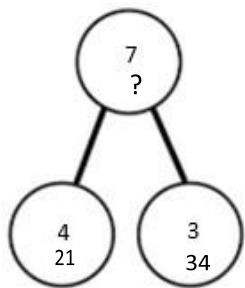
Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

Conceptual variation; different ways to ask children to solve
21 + 34

Word problems:



?	
21	34

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

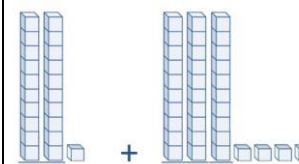
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

$$\boxed{} = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

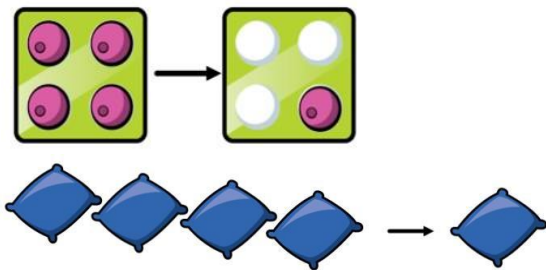
10s	1s
	?
?	5

Calculation policy: Subtraction

Concrete

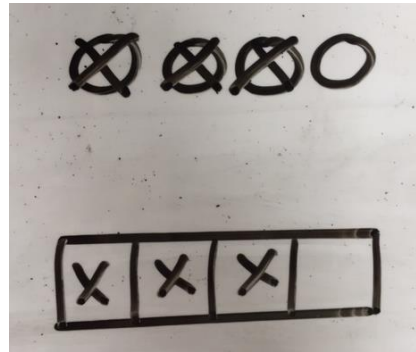
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

$$4 - 3 = 1$$



Pictorial

Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.

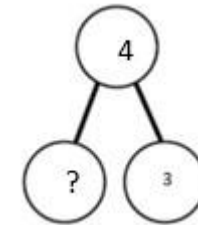


Abstract

$$4 - 3 =$$

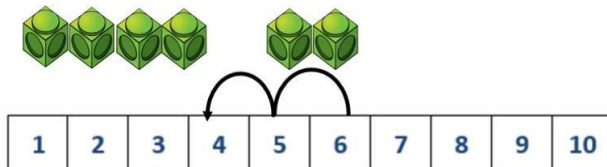
$$\boxed{} = 4 - 3$$

4	
3	?

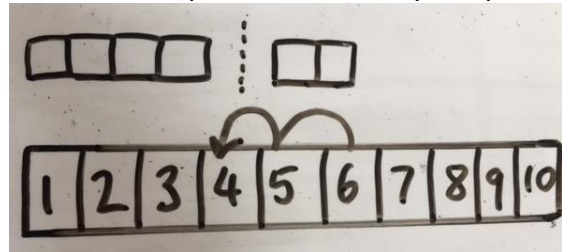


Counting back (using number lines or number tracks) children start with 6 and count back 2.

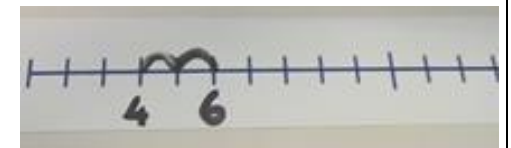
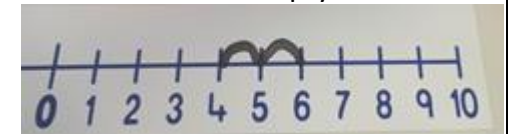
$$6 - 2 = 4$$



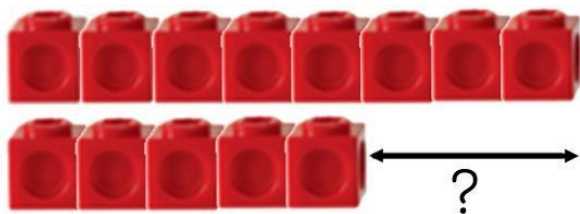
Children to represent what they see pictorially e.g.



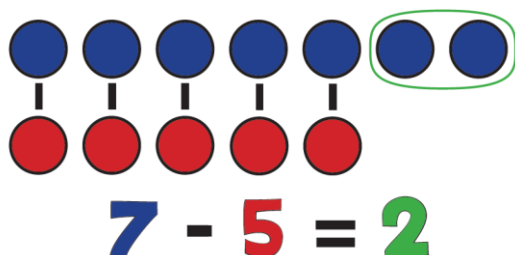
Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line



Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

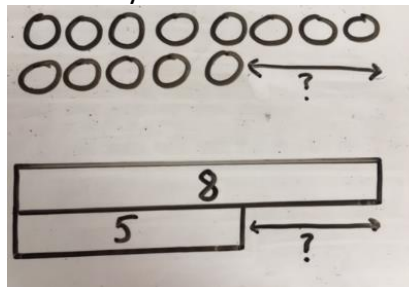


Calculate the difference between 8 and 5.



"How many more is 7 than 5? What is the difference?"

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



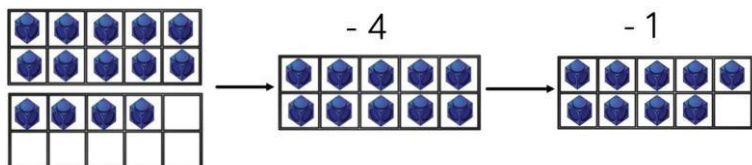
Find the difference between 8 and 5.

8 - 5, the difference is

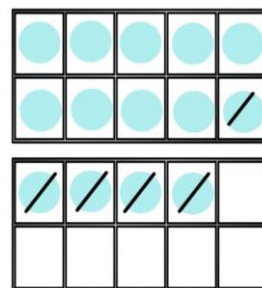
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$
 have the same difference.

Making 10 using ten frames.

$14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



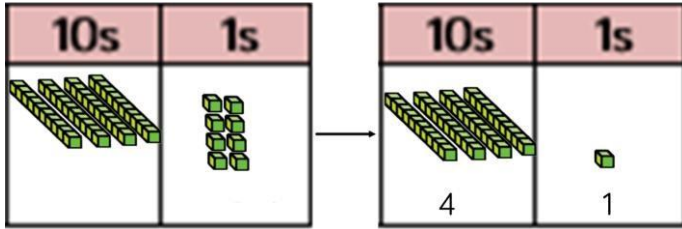
Children to show how they can make 10 by partitioning the subtrahend.

$$\begin{array}{r} 14 - 5 = 9 \\ \swarrow \quad \searrow \\ 4 \quad \quad 1 \end{array}$$

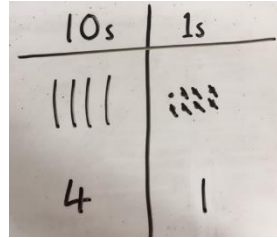
$$\begin{array}{l} 14 - 4 = 10 \\ 10 - 1 = 9 \end{array}$$

Column method using base 10.

48-7



Children to represent the base 10 pictorially.



Column method or children could count back 7.

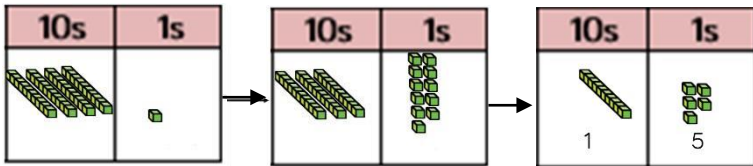
	4	8
-		7
	4	1

68 70 75

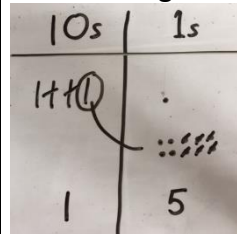
-2 -5

75 - 7 = 68

Column method using base 10 and having to exchange.
41 - 26



Represent the base 10 pictorially, remembering to show the exchange.



Formal column method. Children must understand that when they have exchanged the 10 they still have 75 because $75 = 60 + 15$.

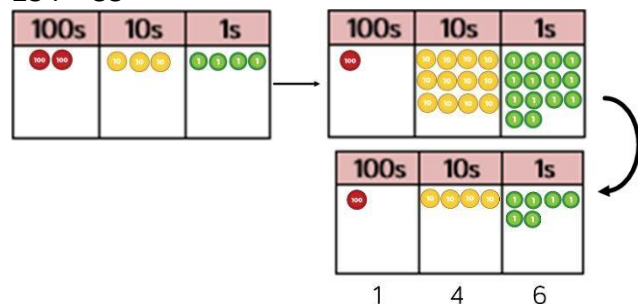
⁶7¹5

- 37

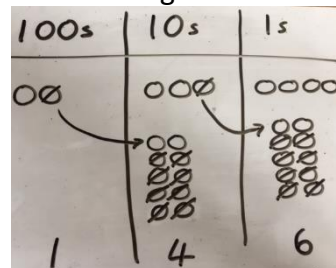
38

Column method using place value counters.

$$234 - 88$$



Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

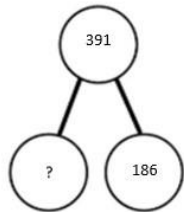
$$\begin{array}{r} 4 1 13 1 \\ \cancel{5} \cancel{0} \cancel{4} \cancel{2} \\ - 1776 \\ \hline 3266 \end{array}$$

Conceptual variation; different ways to ask children to solve
391 - 186

Missing digit calculations

Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups, array, commutative, factor, multiple, multiplicand, scaling .



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array} = 391 - 186$$

What is 186 less than 391?

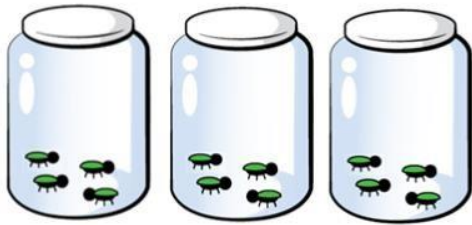
$$\begin{array}{r} 39\Box \\ -\Box\Box6 \\ \hline \Box05 \end{array}$$

Concrete

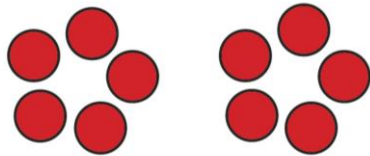
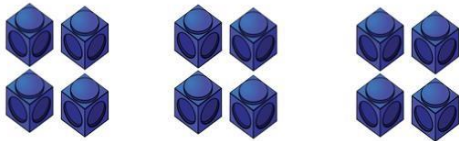
Repeated grouping/repeated addition

$$3 \times 4$$

$$4 + 4 + 4$$



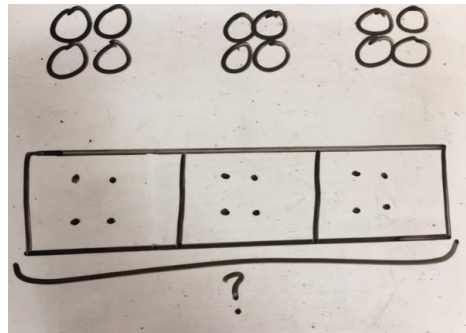
There are 3 equal groups, with 4 in each group.



"2 groups of 5 counters makes 10 counters altogether"

Pictorial

Children to represent the practical resources in a picture and use a bar model.



Abstract

$$3 \times 4 = 12$$

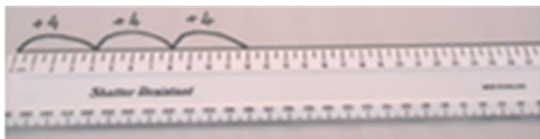
$$4 + 4 + 4 = 12$$



$$5 \times 3 = 5 + 5 + 5 = 15$$

"5 multiplied by 3" means "5, 3 times", which gives "3 lots of 5!"

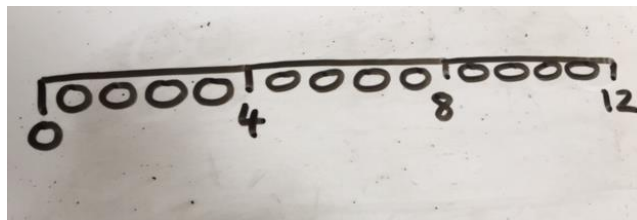
Number lines to show repeated groups-



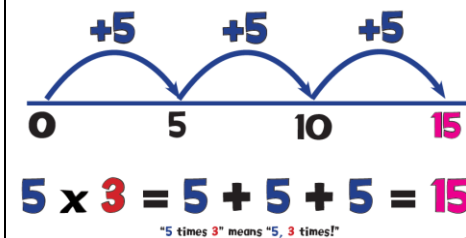
4×3

Cuisenaire rods can be used too.

Represent this pictorially alongside a number line
e.g.:



Abstract number line showing three jumps of five.

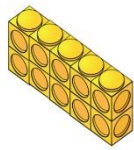


Use arrays to illustrate commutativity counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$

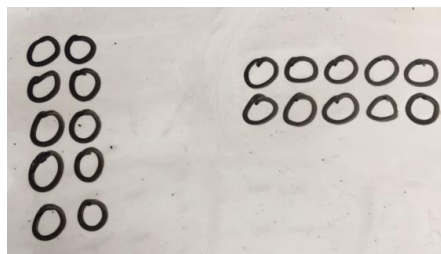


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

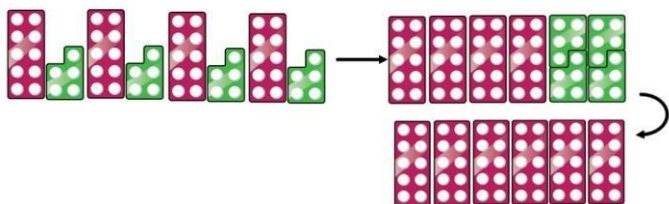
$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

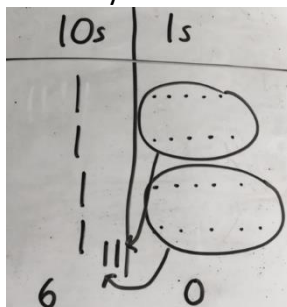
$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$4 \times 15$$

$$\swarrow \searrow$$

$$10 \quad 5$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

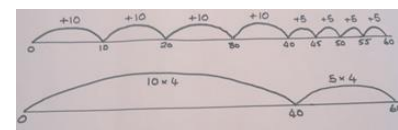
$$40 + 20 = 60$$

$$15 \times 5 = 75$$

$$10 \times 5 = 50$$

$$5 \times 5 = 25$$

$$50 + 25 = 75$$

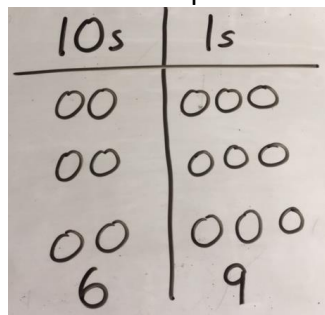


A number line can also be used

Formal column method with place value counters (base 10 can also be used.) 3×23

10s	1s
6	9

Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

$$3 \times 23$$

$$3 \times 20 = 60$$

$$3 \times 3 = 9$$

$$60 + 9 = 69$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Grid method

$$43 \times 6 = 258$$




x	40	3
6	240	18

$$240 + 18 = 258$$

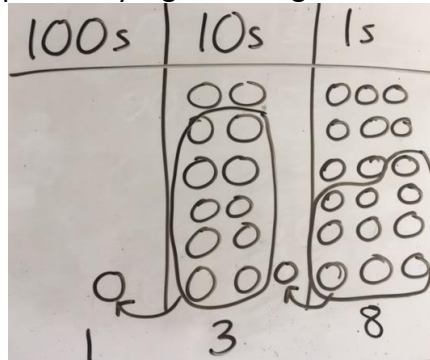
Formal column method with place value counters. 6×23

100s	10s	1s
		



100s	10s	1s
		

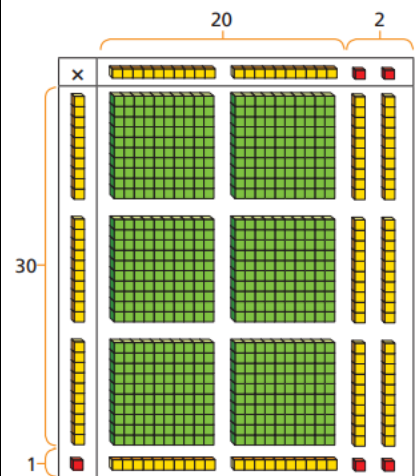
Children to represent the counters/base 10, pictorially e.g. the image below.



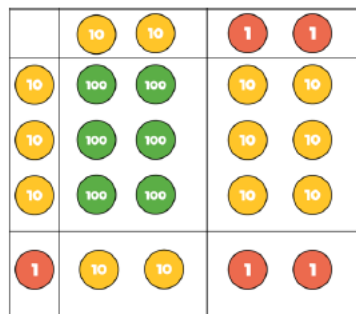
$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$$

Formal written method



Area model



$$22 \times 31 = 682$$

	H	T	O
		2	2
x		3	1
		2	2
	6	6	0
	6	8	2

$$2,739 \times 28 = 76,692$$

TTh	Th	H	T	O
	2	7	3	9
×			2	8
<hr/>				
2	1	9	1	2
₂	₅	₃	₇	
5	4	7	8	0
₁		₁		
7	6	6	9	2

1

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .

To get 2480 they have solved 20×124 .

$$\begin{array}{r}
 1 \ 2 \ 4 \\
 \times \quad 2 \ 6 \\
 \hline
 7 \ 4 \ 4 \\
 2 \ 4 \ 8 \ 0 \\
 \hline
 3 \ 2 \ 2 \ 4 \\
 1 \ 1
 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23

23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$



Find the product of 6 and 23

$$6 \times 23 =$$

$$\boxed{} = 6 \times 23$$

$$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

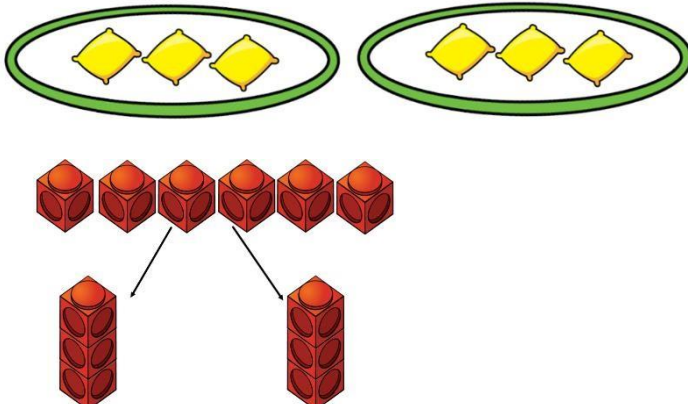
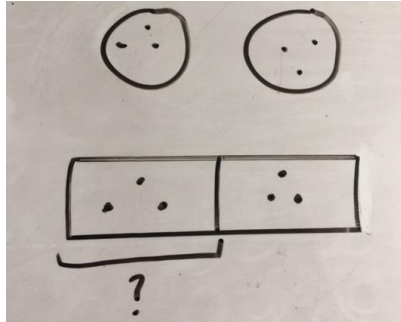
What is the calculation? What

100s	10s	1s
		

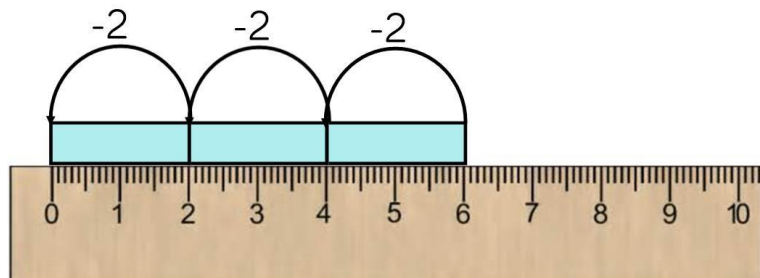
is the product?

Calculation policy: Division

Key language: share, group, divide, divided by, half dividend, divisor, quotient, remainder, scaling

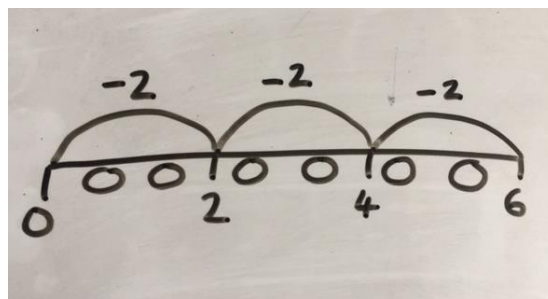
Concrete	Pictorial	Abstract		
<p>Sharing using a range of objects. $6 \div 2$</p>  <p>The concrete representation shows two green ovals, each containing three yellow diamonds, representing the dividend being divided into two equal groups. Below this, six red cubes are arranged in a single row. Two arrows point from the first and fourth cubes to two separate vertical stacks of three cubes each, illustrating the division of the total into two equal groups of three.</p>	<p>Represent the sharing pictorially.</p>  <p>The pictorial representation shows two hand-drawn circles, each containing three dots, representing the dividend being divided into two equal groups. Below the circles, a rectangle is divided into two equal halves by a vertical line. Each half contains three dots. A bracket is drawn under the left half, with a question mark below it, indicating the unknown quotient.</p>	<p>$6 \div 2 = 3$</p> <table><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			

Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$

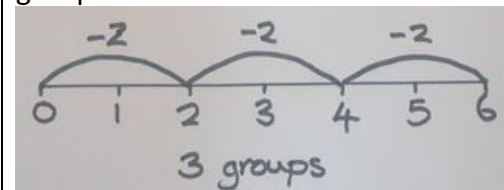


3 groups of 2

Children to represent repeated subtraction pictorially.



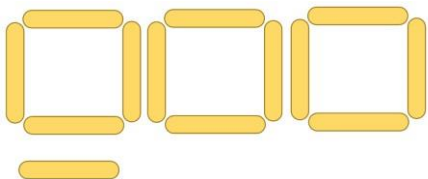
Abstract number line to represent the equal groups that have been subtracted.



$2d \div 1d$ with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

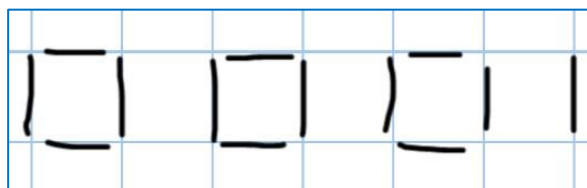
$$13 \div 4$$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

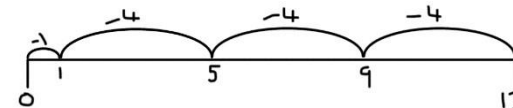


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

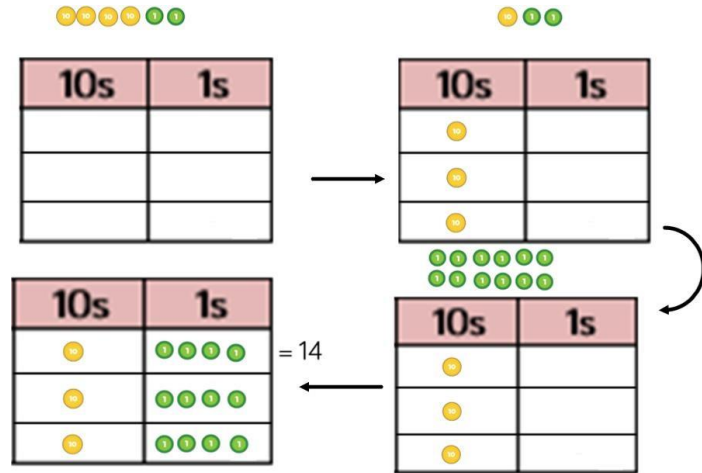
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

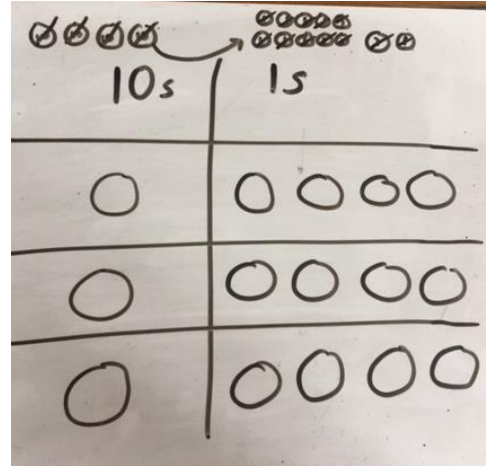


Sharing using place value counters.

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.

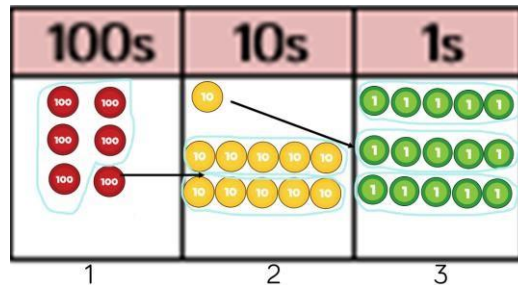


Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

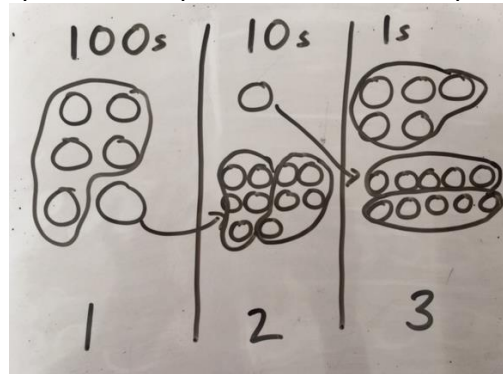
Short division using place value counters to group.

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

72 ÷ 4 = 18

$$\begin{array}{r} 18 \\ 4 \overline{) 72} \\ \underline{4} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

		0	3	6
	12	4	⁴ 3	⁷ 2

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	⁷ 3	¹³ 3	¹³ 5

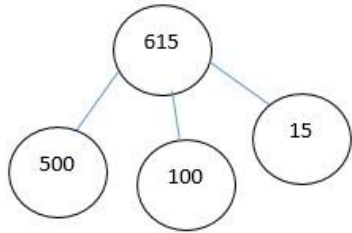
15	30	45	60	75	90	105	120	135	150
----	----	----	----	----	----	-----	-----	-----	-----

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

$$37 \overline{) 983} \begin{matrix} 26 \\ \text{r}21 \end{matrix}$$

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation? What is the answer?

